From quantum to quantales and beyond

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Abstract

The axioms of quantum mechanics laid out by von Neumann involved Hilbert spaces and operators on them, and it soon became apparent that the properties of collections of operators are important, as evidenced by the extensive study of rings of operators (later called von Neumann algebras) in the famous series of papers by Murray and von Neumann. An abstraction of these is given by C*-algebras, which have developed into an area of mathematics of their own, with many applications not necessarily related to their origins in quantum theory. For instance, abelian subalgebras of a C*-algebra are relevant in algebraic quantum mechanics because they relate to the notion of a classical observer, but they also arise naturally when studying C*-algebras associated to discrete dynamical systems. Although not being part of the standard toolkit of a quantum physicist, quantales can be regarded as another spin-off of quantum theory, moreover one that is also firmly anchored in pointfree topology and category theory. In this talk I shall describe recent developments in the interplay between C*-algebras and quantales that hinge on abelian subalgebras and lead to interesting philosophical connections to the foundations of quantum mechanics and to the ever controversial role of consciousness in them. Groupoids and inverse semigroups feature prominently, in particular in the context of "C*-algebras over groupoids" (Fell bundles), and hints on how to proceed follow from keeping in mind a modicum of categorical thinking.