

BOUNDEDNESS OF STEIN'S SPHERICAL MAXIMAL FUNCTION IN VARIABLE LEBESGUE SPACE AND APPLICATION TO THE WAVE EQUATION

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Abstract. Define the Spherical Maximal operator \mathcal{M} , by

$$\mathcal{M}f(x) := \sup_{t>0} |\mu_t * f(x)|,$$

where μ_t denotes the normalized surface measure on the sphere of center 0 and radius t in \mathbb{R}^n .

We show sufficient conditions of Hölder type on $p(\cdot)$ with $\frac{n}{n-1} < p^- \leq p^+ \leq p^-(n-1)$ such that

$$\|\mathcal{M}f\|_{p(\cdot)} \lesssim \|f\|_{p(\cdot)}.$$

The result is then interpreted as the preservation of the integrability properties of the initial velocity of propagation to the solution of the initial-value problem for the wave equation.