

# Geometry of numerical range of linear pencils

Fatemeh Esmaeili Taheri<sup>1</sup> Joint work with  
Natalia Bebiano<sup>1</sup>

<sup>1</sup> *CMUC - Centro de Matemática da Universidade de Coimbra,  
Portugal*

## Abstract

Let  $A, B$  be  $n \times n$  (complex) matrices. The numerical range of the linear pencil  $A - \lambda B$  where  $\lambda$  is a complex number, is the set

$$W(A, B) = \{x^*(A - \lambda B)x : x \in \mathbb{C}^n, \|x\| = 1, \lambda \in \mathbb{C}\}.$$

We are mainly interested in the study of the numerical range of a linear pencil,  $A - \lambda B$ , when one of the matrices  $A$  or  $B$  is Hermitian and  $\lambda \in \mathbb{C}$ .

In this talk, the geometrical properties of  $W(A - \lambda B)$ , with emphasis to its boundary are presented.

We characterize  $W(A, B)$  for matrices of small dimensions in terms of certain algebraic curves. The results are illustrated by numerical examples.

## Keywords

Numerical range, Linear pencil, Generalized eigenvalue problem, Plane algebraic curve.

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