On finite simple quotients of triangle groups

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Abstract. Given a triple (a, b, c) of positive integers, a finite group is said to be an (a, b, c)-group if it is a quotient of the triangle group

$$T_{a,b,c} = \langle x, y, z : x^a = y^b = z^c = xyz = 1 \rangle.$$

Let $G_0 = G(p^r)$ be a finite quasisimple group of Lie type with corresponding simple algebraic group G. Given a positive integer a, let $G_{[a]} = \{g \in G : g^a = 1\}$ be the subvariety of G consisting of elements of order dividing a, and set $j_a(G) = \dim G_{[a]}$. Given a triple (a, b, c) of positive integers, we conjectured a few years ago that if $j_a(G) + j_b(G) + j_c(G) = 2 \dim G$ then given a prime p there are only finitely many positive integers r such that $G(p^r)$ is an (a, b, c)-group. We present some recent progress on this conjecture and related results: in particular the conjecture holds for finite simple groups.