The Gelfand problem for the Infinity Laplacian

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Abstract

We study the asymptotic behavior as $p \to \infty$ of solutions to the p-Laplacian Gelfand-type problem

$$\begin{cases} -\Delta_p u = \lambda\, e^u & \text{in } \Omega \subset \mathbb{R}^n \\ \quad u = 0 & \text{on } \partial\Omega. \end{cases}$$

We identify a precise scaling between u and the bifurcation parameter λ that balances reaction and diffusion and produces a nontrivial limit problem. More precisely, under an appropriate rescaling on u and λ, we prove uniform convergence of solutions to the p-Laplacian Gelfand-type problem to solutions of

$$\begin{cases} \min\left\{|\nabla u| - \Lambda\, e^u, -\Delta_\infty u\right\} = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

We discuss existence, non-existence, and multiplicity of solutions of the limit problem in terms of the limit bifurcation parameter.

Moreover, we prove a comparison principle for small solutions of the limit equation. This result is interesting for two main reasons. Firstly, the limit equation is not proper, a basic requirement for comparison. Secondly, based on the multiplicity results for the p-Laplacian Gelfand-type problem in the literature, one cannot expect comparison to hold in general. The key idea is a change of variables that allows us to obtain a proper equation for solutions with $\|u\|_\infty < 1$. Remarkably, minimal solutions of the limit equation verify this condition, and we can conclude they are the only ones with $\|u\|_\infty < 1$. To the best of our knowledge, this result has no counterpart for $p < \infty$.

This is a joint work with B. Son (University of Maine) and P. Wang (Wayne State University).