ENRICHED PRIESTLEY SPACES

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Priestley spaces [5, 6], the duals of distributive lattices, are by definition those partially ordered compact spaces X where the cone

$$(f\colon X \to 2)_f$$

of morphisms into the two-element space is point-separating and initial. In our recent study of Stone-type dualities [4], we extended the context from order structures to metric structures and in particular went from ordered compact Hausdorff spaces to metric compact Hausdorff spaces. This step led naturally to the notion of *metric Priestley space*: those metric compact Hausdorff spaces X where the cone

$$(f\colon X \to [0,1]^{\mathrm{op}})_f$$

of all morphisms into the unit interval is point-separating and initial. We note that, in the metric setting, every partially ordered compact space is Priestley, and so is every classic compact metric space. Motivated by classic results about compact Hausdorff spaces and more recent results about partially ordered compact spaces [3, 1, 2], in this talk we investigate the algebraic character of the dual of the category of metric (and, more general, quantale enriched) Priestley spaces and morphisms.

References

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