Limiting embeddings in smoothness Morrey spaces, and applications

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The classical Morrey spaces $\mathcal{M}_{u,p}$, $0 < p \leq u < \infty$, were introduced by Ch.B. Morrey (1938) and are part of the wider class of Morrey-Campanato spaces. They contain all locally p-integrable functions f such that

$$\|f|\mathcal{M}_{u,p}(\mathbb{R}^n)\| = \sup_{x\in\mathbb{R}^n, R>0} R^{\frac{n}{u}-\frac{n}{p}} \left(\int_{B(x,R)} |f(y)|^p \,\mathrm{d}y \right)^{1/p}$$

is finite, where $B(x,R)$ are the usual balls centered at $x \in \mathbb{R}^n$ with radius $R > 0$. In view of $L_u(\mathbb{R}^n) = \mathcal{M}_{u,u}(\mathbb{R}^n) \hookrightarrow \mathcal{M}_{u,p}(\mathbb{R}^n)$ for any $p \leq u$ they are considered as an extension of the scale of L_p spaces. Built upon these basic spaces Besov-Morrey spaces $\mathcal{N}^s_{u,p,q}$ and Triebel-Lizorkin-Morrey spaces $\mathcal{E}^s_{u,p,q}$ attracted some attention in the last years, in particular in connection with Navier-Stokes equations. Though a lot has been done recently, almost nothing is known about properties of embeddings of these spaces and corresponding applications to spectral theory.

We characterise continuous embeddings of spaces of Besov-Morrey type,

$$\mathcal{N}^{s_1}_{u_1,p_1,q_1}(\mathbb{R}^n) \hookrightarrow \mathcal{N}^{s_2}_{u_2,p_2,q_2}(\mathbb{R}^n),$$

always assuming that $s_i \in \mathbb{R}$, $q_i \in (0,\infty]$, $0 < p_i \leq u_i < \infty$, $i = 1,2$, and their counterparts in the Triebel-Lizorkin scale. In particular, we focus on the limiting situation when $s_1 - \frac{n}{u_1} = s_2 - \frac{n}{u_2}$ in the above setting.

Closely related to these scales are the spaces of Besov-type $B^{s,\tau}_{p,q}$ and of Triebel-Lizorkin type $F^{s,\tau}_{p,q}$, $\tau \geq 0$, which coincide with their classical counterparts for $\tau = 0$. We can also characterise embeddings within these scales of spaces.

As some application we study the continuity envelopes $\mathfrak{E}_{\mathsf{C}}(X) = \left(\mathcal{E}^X_{\mathsf{C}}, u^X_{\mathsf{C}}\right)$ of the above spaces, and obtain some surprising characterisations. We shall also discuss some applications to Hardy-type inequalities and approximation numbers.

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