Compact embeddings of weighted Sobolev spaces

Dipl. - Math. Therese Mieth

(Friedrich Schiller University Jena, Germany)

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Abstract

Let B be the unit ball in $\mathbb{R}^n$ and ψ be a continuous admissible function on $(0,1]$ with $\psi(1) = 1$ and bounded from below.
We define the weighted Sobolev space $E^m_{p,\psi}(B)$, $1 \leq p < \infty, m \in \mathbb{N}$, as the completion of $C^m_0(B) = \{f \in C^m(B) : \operatorname{supp} f \text{ compact}\}$ in the norm

$$\|f \ | E^m_{p,\psi}(B)\| := \left(\int_B |x|^{mp} \ \psi^p(|x|) \sum_{|\alpha|=m} |\mathrm{D}^\alpha f(x)|^p \mathrm{d}x \right)^{1/p}.$$

We consider the continuous embedding

$$\mathrm{id} : E^m_{p,\psi}(B) \hookrightarrow L_p(B).$$

This talk gives some general assertions about the compactness of id. Furthermore, asking for the quality of compactness measured in terms of entropy and approximation numbers, we can give precise statements on the asymtoptic behaviour in case of $p = 2$. Therefore, specific Hilbert space arguments from operator theory and the distribution of eigenvalues of some degenerate elliptic operators are used.