A class of electromagnetic p(x,t)-curl systems: existence and uniqueness, blow-up and finite time extinction

S.N. Antontsev
CMAF-CIO, University of Lisbon, Portugal
Institute of Hydrodynamics, Novosibirsk, Russia

We consider in $Q_T = \Omega \times (0,T), \Omega \subset \mathbb{R}^3$, the following class of $p(x,t)$-curl systems arising in electromagnetism

$$\partial_t \boldsymbol{h} + \nabla \times \left(|\nabla \times \boldsymbol{h}|^{p(x,t)-2} \nabla \times \boldsymbol{h} \right) = \boldsymbol{f}(\boldsymbol{h}), \quad \nabla \cdot \boldsymbol{h} = 0 \quad \text{in } Q_T, \tag{1}$$

$$|\nabla \times \boldsymbol{h}|^{p(x,t)-2} \nabla \times \boldsymbol{h} \times \boldsymbol{n} = \boldsymbol{0}, \quad \boldsymbol{h} \cdot \boldsymbol{n} = 0 \text{ on } \Sigma_T, \tag{2}$$

$$\boldsymbol{h}(\,\cdot\,, 0) = \boldsymbol{h}_0 \text{ in } \Omega. \tag{3}$$

where $\boldsymbol{h}$ is the unknown magnetic field and $\boldsymbol{h}_0$ is a given function. The given log -continuous function $p(x,t)$ satisfies $\frac{6}{5} < p^- \le p(x,t) \le p^+ < \infty$. The nonlinear function $\boldsymbol{f}(\boldsymbol{h})$ can model either a source term of the type $\boldsymbol{f}(\boldsymbol{h}) = \boldsymbol{h} \left(\int_\Omega |\boldsymbol{h}|^2 \right)^{\frac{\sigma-2}{2}}$, with $\sigma > 1$ or a sink term $\boldsymbol{f}(\boldsymbol{h}) = -\boldsymbol{h} \left(\int_\Omega |\boldsymbol{h}|^2 \right)^{-\lambda}$, with $\lambda > 0$. We prove existence and uniqueness of solution for (1)-(3).

Blow-up of local solutions is studied in the case of source term and finite time extinction of solution is proved in the case of sink term. The detailed proofs can be found in [1-3]. (Joint work with Miranda F. and Santos L.)

References

1. Antontsev S. N., Shmarev S.I., Evolution PDEs with nonstandard growth conditions. Existence, Uniqueness, Localization, Blow-up, Series: Atlantis Studies in Differential Equations, Vol. 4, XIV, 393 p. Atlantis Press, Paris, France, 2015.

2. Antontsev S. N., Miranda. S., Santos L., A class of electromagnetic p-curl systems: blow-up and finite time extinction, Nonlinear Analysis: Theory, Methods, Applications, **75**, 3916-3929(2012).

3. Antontsev S. N., Miranda. S., Santos L., Blow-up and finite time extinction for p(x,t)-curl systems arising in electromagnetism, J. of Math. Analysis and Applications, **440**, 300-322(2016).