Special bi-invariant connections and finite dimensional Poisson algebras

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Let G be a connected Lie group and $\mathcal{G}$ its Lie algebra. We denote by ∇^0 the torsion free bi-invariant linear connection on G given by $\nabla^0_X Y = \frac{1}{2}[X,Y]$, for any left invariant vector fields X, Y. A Poisson structure on the Lie algebra $\mathcal{G}$ of G is a commutative and associative product on $\mathcal{G}$ for which ad_u is a derivation, for any $u \in \mathcal{G}$. A torsion free bi-invariant linear connections on G which have the same curvature as ∇^0 are called special. We show that there is a bijection between the space of special connections on G and the space of Poisson structures on $\mathcal{G}$. We compute the holonomy Lie algebra of a special connection and we show that the Poisson structures associated to special connections which have the same holonomy Lie algebra as ∇^0 possess interesting properties. Finally, we study Poisson structures on a Lie algebra and we give a large class of examples which gives, of course, a large class of special connections.