# THE RIEMANN-HILBERT PROBLEM APPLIED TO THE THEORY OF ORTHOGONAL POLYNOMIALS

VÍTOR LUÍS SOUSA *

## Abstract

We considered orthogonal polynomials with respect to the absolutely continuous measures, with $x_0$ having both a step-like and an algebraic singularities, of the following type:

$$|x_0 - x|^\gamma \times \begin{cases} 1, & \text{for } x \in [-1, x_0)\,, \\ c^2, & \text{for } x \in [x_0, 1]\,, \end{cases} \quad (c > 0,\ \gamma > -1,\ x_0 \in (-1, 1)).$$

At $x_0$, using the results obtained for the strong uniform asymptotic behavior of the orthogonal polynomials it obtains a new reproducing kernel in terms of Confluent Hypergeometric functions, in the case $\gamma \neq 0$ with $c \neq 1$, which is a generalization of the called second Bessel kernel. It provides the first explicit example of a reproducing kernel that belongs in the De Branges space different of those of the classic Paley-Wiener, as well as, taking the step-like discontinuity ($\gamma = 0$, $c \neq 1$), an explicit example of violation of the universality even when weight is nonzero. The technique that allows us the proof of the results obtained is based on the characterizing the orthogonal polynomials in terms of a Riemann-Hilbert problem, and, to use the nonlinear steepest descent method, introduced by Deift and his collaborators.

* Escola Secundária João da Silva Correia and
Center for Research and Development in Mathematics and Applications - University of Aveiro,
email: `vsousa@ua.pt`