# Volterra integral equations for non linear partial differential equations of integrable type

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Nonlinear partial differential equations (NPDE) of integrable type have important physical applications. They are used to describe electromagnetic waves in optical fibers, surface wave dynamics, charge density waves, breaking wave dynamics etc. Their corresponding initial value problem (IVP) can be solved with the help of the so-called Inverse Scattering Transform (IST) method (see, for instance, [3]) which allows us to solve it in three steps by only using the initial potential.

In this talk we focus on the IVP associated to a specific NPDE of integrable type that is the Korteweg-de Vries (KdV) equation which governs the propagation of surface water waves in long, narrow, shallow canals [2]

$$\begin{cases} \dfrac{\partial q(t,x)}{\partial t} - 6q(t,x)\dfrac{\partial q(t,x)}{\partial x} + \dfrac{\partial^3 q(t,x)}{\partial x^3} = 0, \qquad x \in \mathbb{R}, \qquad t \in \mathbb{R}^+ \\ q(0,x) = q(x). \end{cases}$$

We present a numerical method to solve the first step of the path of the IST method [1]. It consists of approximating the solution of the following Volterra integral equations:

$$K(x,x+y) - \frac{1}{2}\int_0^y \int_{x+\frac{1}{2}(y-x)}^{\infty} q(t)K(t,t+s)dt\,ds = \frac{1}{2}\int_{x+\frac{1}{2}y}^{\infty} q(z)dz,$$

$$x \in \mathbb{R}, \quad y \geq 0$$

$$M(x,x-y) - \frac{1}{2}\int_0^y \int_{-\infty}^{x-\frac{1}{2}(y-s)} q(t)\,M(t,t-s)\,dt\,ds = \frac{1}{2}\int_{-\infty}^{x-\frac{1}{2}y} q(z)\,dz,$$

where $q \in L^1(\mathbb{R};(1+|x|)dx)$ and $K$ and $M$ are bivariate unknown functions.

The numerical procedure is based on some basic properties of the unknown functions that we prove. These properties allow us to know $K$ and $M$ in each point of their unbounded domains by solving the equations in a bounded triangle. For this reason, we developed a particular technique to solve them in this computational triangle. The results give numerical evidence of its effectiveness.

This is joint work with Cornelis van der Mee and Sebastiano Seatzu.

## References

[1] L. Fermo, C. van der Mee and S. Seatzu, A Numerical Method for Volterra Integral Equations Basic to the Solution of the KdV Equation, submitted (2017).

[2] D. J. Korteweg and G. de Vries, On the change of form of long waves advancing in a rectangular channel and on a new type of long stationary waves, Phil. Mag. 39, 422-443 (1895).

[3] C. van der Mee, Nonlinear Evolution Models of Integrable Type, e-Lecture Notes SIMAI, Vol. 11, 2013.