TITLE: *The Universal Generalization Problem: Euclid's Mathematical Solutions and Contemporary Formal Accounts*

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ABSTRACT:
The goal of my talk is to analyze different contemporary foundations of Euclid's Geometry[1], compare them with each other and with the philological studies of ancient texts. In particular, I will analyze how these foundational studies fare with respect to the *Universal Generalization Problem* (UGP). This problem boils down to the following question: What entitles one to pass from particular premises to a general conclusion?, or What entitles one to conclude that a property which holds for a specific object also holds for any object of the same kind?
Consider, for example, Euclid's proof of Proposition I.32: "In any triangle, the interior angles are equal to two right angles". Euclid begins as follows: Let ABC be a triangle. Then he shows that the interior angles of ABC are equal to two right angles. Euclid carries out his proof on an individual triangle ABC, and then he concludes that the property established for ABC holds for all triangles. What entitles Euclid to do this?
The Universal Generalization Problem is well known –from ancient times- to philosophers and mathematicians as well, and this makes it a perfect test bench for evaluating the contemporary foundations of Euclid's geometry. While on the one hand it allows us to assess the faithfulness of these foundations to Greek mathematics, on the other, it enables us to evaluate these formal accounts of Euclid's *Elements* as foundations of Euclidean geometry per se, highlighting the logical formalization of particular mathematical processes.
These comparisons will show that different contemporary formal accounts of Euclid's *Elements* give different answers to the UGP because these answers are inspired by different philological interpretation of Euclid's text. Moreover, these comparisons will highlight how the issue of the UGP in Euclid is more complex than it is often suggested by the extant formal accounts of Euclid's geometrical thinking.

1 Mueller [1981]; Mäenpää, von Plato [1990]; Mäenpää [1993; 1997]; Mumma [2006]; Graziani [2007]; Miller [2008]; Mumma, Avigad, Dean [2009]; Beeson [2009; ms].