Centro de Matemática
Universidade de Coimbra

Linha de Álgebra e Combinatória

Wednesday, June 5th 2019 at 14:30

Samuel Lopes
(CMUP, Universidade do Porto)

Normally ordered forms of powers of differential operators

Given a derivation ∂ of a ring, and a central element h in this ring, we can consider the normally ordered forms of the powers $(h\partial)^n$ of the differential operators $h\partial$. For instance,

$$\begin{aligned}(h\partial)^2 &= hh'\partial + h^2\partial^2,\\ (h\partial)^3 &= h(h')^2\partial + h^2h''\partial + 3h^2h'\partial^2 + h^3\partial^3.\end{aligned}$$

It is known that these formulas can be described nicely in terms of sums over increasing trees. Also, they include, either as coefficients or as restricted sums of coefficients, well–known families of combinatorial numbers (namely, the Stirling numbers of both kinds, and the Euler numbers).

We extend this study to the more general operators $(h^d\partial)^n$. In particular, we give a simple combinatorial interpretation for the coefficients of their normally ordered forms. As a byproduct, we obtain new formulas for the *generalized Stirling numbers* $\left\{ {n \atop k} \right\}_{q,d}$ (coefficients of the normally ordered form of $(x^q\partial^d)^n$ in the Weyl algebra):

$$\left\{ {n \atop k} \right\}_{q,d} = \sum_A \frac{\prod_i (d)_{r_i(A)} \cdot \prod_j (q)_{c_j(A)}}{\prod_{i,j}(a_{i,j}!)}$$

where the sum is carried over all triangular arrays $A = (a_{i,j})_{1\le j<i\le n}$ of nonnegative integers, the $r_i(A)$ and $c_j(A)$ being the row sums and column sums of A, and $(x)_k$ standing for the falling factorial.

We also study the behaviour of the aforementioned coefficients modulo prime powers, generalizing several known results for Stirling numbers of both kinds and Euler numbers.

This is joint work with E. Briand and M. Rosas (U. Sevilla, Spain).

COMPETE QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL UNIÃO EUROPEIA Fundo Europeu de Desenvolvimento Regional FCT Fundação para a Ciência e a Tecnologia MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR