THE POINTFREE REPRESENTATION OF A TRUNCATED ARCHIMEDEAN ℓ-GROUP

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In this talk we develop the analog for truncated archimedean lattice-ordered groups, hereafter referred to as truncs, of Madden's pointfree representation for **W**, the category of archimedean ℓ-groups with designated weak order unit. In the first part of the talk we will motivate the notion of truncation with a few sketches, review its definition, and outline the pointed, or Yosida representation of truncs. We shall point out the necessity of using pointed spaces and maps in order to render the representation functorial.

In the second part of the talk we will discuss the main result, which is the pointfree representation of truncs.

Theorem 1. *For every archimedean trunc* A *there is a regular Lindelöf pointed frame* L*, a subtrunc* $\widehat{A}$ *of* $\mathcal{R}_0 L$*, and a trunc isomorphism* $A \to \widehat{A}$*. The pointed frame* L *is unique with respect to its properties, and the representation is functorial.*

A pointed frame is just a (completely regular) frame L equipped with a designated point $* : L \to 2$; for example, the pointed frame of the reals, designated $\mathcal{O}_0\mathbb{R}$, is just the topology $\mathcal{O}\mathbb{R}$ of the real numbers equipped with the frame map $\mathcal{O}\mathbb{R} \to 2$ corresponding to the insertion $0 \to \mathbb{R}$. A pointed frame map is just a frame map which commutes with the designated points, and $\mathcal{R}_0 L$ stands for the trunc of pointed frame maps $\mathcal{O}_0\mathbb{R} \to L$.

If time permits we shall briefly discuss several beckoning directions for extending the rich theory of **W** to truncs. Examples include uniform approximation and completion, and the characterization of epimorphisms.

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