Mixed-Type Functional Differential Equations:
Analysis and Computational Methods

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In the present talk we are concerned with the approximate solution of functional differential equation with both delayed and advanced arguments. Such equations are often referred in the literature as mixed type functional differential or forward-backward equations. The analysis of this type of equation is recent and some important questions remain open.

We begin by considering the equation

$$x'(t) = \alpha(t)x(t) + \beta(t)x(t-1) + \gamma(t)x(t+1). \quad (1)$$

One looks for a solution x, defined for $t \in [-1, k]$,($k \in \mathbf{N}$), that satisfies this equation almost everywhere on $[0, k-1]$ and assumes specified values on the intervals $[-1, 0]$ and $(k-1, k]$, ($k \in \mathbf{N}$).

This problem was studied in [1], [2] and [3] (among other articles). In those papers, the question of existence and uniqueness of solution was addressed and different computational methods were applied to its numerical approximation. It was remarked that one of the main difficulties of this problem results from its ill-conditioning. Even for not very high values of k, the numerical methods become unstable.

More recently, we have started to study a modification of equation (1), including a nonlinear term, depending on $x(t)$. In this case, we search for a solution that is defined on the whole real axis and tends to given values as t tends to $\pm\infty$. This nonlinear problem arises from a mathematical model in nerve conduction. In the present talk, we will give the outline of the method, proposed for its solution.

The talk is based on a joint work with Filomena Teodoro (CEMAT/IST), Neville Ford and Patricia Lumb (Dep. Mathematics, University of Chester).

References

[1] N. J. Ford and P. M. Lumb, *Mixed-type functional differential equations: a numerical approach*, J. Comput. Appl. Math., 229(2009), 471-479.

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