

Abstract

We will discuss the entropy numbers of compact Sobolev embeddings of spaces that consist of radial and block-radial functions. The numbers described the compactness of the embeddings in a quantitative way that can be used in the spectral theory of linear operators.

More precisely, Let $SO(\gamma) = SO(\gamma_1) \times \dots \times SO(\gamma_m)$ be a group of block-radial symmetries in $\mathbb{R}^d = \mathbb{R}^{\gamma_1} \times \dots \times \mathbb{R}^{\gamma_m}$, $\gamma = (\gamma_1, \dots, \gamma_m)$. Let $R_\gamma H_p^s(\mathbb{R}^d)$ be a subspace of the Sobolev space $H_p^s(\mathbb{R}^d)$ that consists of block-radial functions i.e. functions invariant with respect to the action of the group $SO(\gamma)$. We prove that the asymptotic behaviour of the entropy numbers of compact embeddings $\text{id} : R_\gamma H_{p_1}^{s_1}(\mathbb{R}^d) \hookrightarrow R_\gamma H_{p_2}^{s_2}(\mathbb{R}^d)$ depends on the number of equal blocks and dimensions of blocks, the parameters p_1 and p_2 , but is independent of the smoothness parameters s_0, s_1 . We apply the asymptotic behaviour to estimation of power of a negative spectrum of Schrödinger type operators with block-radial potentials. This part essentially relies on the Birman-Schwinger principle.

References

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