

## Congruence modular variety and algebraic crystallography

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In (B-Gran, 2004), was given a categorical formulation of the Shifting Lemma which is a characterization of the Congruence Modular Varieties among all the variety of Universal Algebra, introduced in (Gumm, 1983).

Starting from a characterization of this Shifting Lemma by a property of the fibers of the fibration of points  $\mathfrak{A}\mathbb{E}$  (B, 2005), on the model of what happens for Mal'tsev categories, we shall investigate it in three directions :

- 1) a new one : in following the golden thread of abelian split epimorphisms naturally provided by this characterization ;
- 2) a more or less expected one : in measuring the distance between the consequences of the Shifting Lemma in the varietal context and in the much more general categorical one ;
- 3) a quite amazing phenomenon, which I should call Algebraic Crystallography, and which I am going to briefly describe now.

### Algebraic chrystallography

Let us define as *chrystallographic for a given algebraic structure* any categorical setting in which, on any object  $X$  of this setting, *there is at most one internal algebraic structure of this kind.*

Such kinds of situations are well known from a long time ; for instance, it is clear that in a pointed Jónnson-Tarski variety, on any algebra  $X$  there is at most one internal commutative monoid structure ; the same property holds for the commutative and associative (=autonomous) Mal'tsev operations in the Mal'tsev varieties. The first property still holds for the unital categories and the second one for the Mal'tsev categories.

But, in a way, the particular varietal origin of these first examples lessened the surprise effect : this phenomenon arised naturally because, and when, some term of the variety  $\mathbb{V}$  becomes a homomorphism.

Now, along with some aspects of our investigations, will emerge situations where it is impossible to skip the surprise effect, namely where the algebraic structure in question is no more (at least directly) determined by the terms of the ambient variety.