Context-free word problem semigroups

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The word problem of an algebraic structure is, informally, the problem of determining whether two expressions over a finite generating set represent the same element of the structure. For groups, this is equivalent to determining which words represent the identity, and so the word problem of a group G with respect to a finite generating set X is formally defined as $W(G, X) = \{w \in (X \cup X^{-1})^* \mid w =_G 1\}$. This can then be considered as a formal language (a set of words over a finite alphabet), and there has been much study of the connections between the algebraic properties of groups and the language-theoretic properties of their word problems, with the seminal result in this area being the characterisation by Muller and Schupp of groups with context-free word problem as being precisely the virtually free groups.

There are various ways of formally defining the word problem for semigroups in general. The most-studied version is the 'unfolded word problem' of a semigroup S with respect to a finite generating set X; this is the set $WP(S, X) = \{u\#v^{\mathrm{rev}} \mid u =_S v\}$, where $\#$ is some symbol not in X, and v^{rev} is the reverse of v. I will present recent results on the class of semigroups with context-free (unfolded) word problem, with a particular focus on the extent to which this class is closed under various standard semigroup constructions (direct product, free product etc.), and on semigroups with deterministic context-free word problem, which are of particular interest since all groups with context-free word problem (i.e. virtually free groups) have deterministic context-free word problem. This is joint work with Alan Cain and Markus Pfeiffer, and with Alan Cain and Mark Kambites.

The talk will include a brief introduction to context-free languages.

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