Discrete covers of locales

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If L is a locale, the dual of the lattice S(L) of its sublocales is a zero-dimensional locale, but it is typically non-Boolean. Recently [1] it has been shown that the system $S_c(L)$ consisting of joins of closed sublocales is always a frame, and for L subfit it is a complete Boolean algebra which coincides with the Booleanization of S(L). Therefore, $S_c(L)$ can be seen as a discretization of L (for L subfit), and it has found several applications in point-free topology. In this talk, we will discuss the Booleanization of S(L) for a general L, its close relation to the T_D axiom (in particular, giving a characterization for T_D -spatial locales) and some new aspects of its (non-)functorial behavior.

References

 Picado, J., Pultr, A., Tozzi, A.: Joins of closed sublocales. Houston J. Math. 45, 21–38 (2019).