

ON THE TERNARY COMMUTATOR, I: EXACT MAL'TSEV CATEGORIES

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ABSTRACT

In his study of *polynomially inequivalent algebras*, A. Bulatov introduced in [4] a *higher-order commutator operator* of congruences in Mal'tsev algebras, based on a generalization of the so-called *term condition*. His notion was further developed by E. Aichinger and N. Mudrinski in [1] in the context of Mal'tsev varieties and can be viewed in some sense as a generalization of the *binary Smith commutator* originally introduced by J. D. H. Smith in 1976 [9]. This new concept of *higher-order commutator* has been the subject of several studies in universal algebra [7, 5, 6] and several procedures of defining the higher-order commutator of congruences were found. For congruence modular varieties, it has analogues for those properties shown to be valid for the binary Smith commutator such as monotonicity, stability with respect to joins, stability with respect to restriction, etc.

In categorical algebra, a first generalization of the binary Smith commutator of two equivalence relations on the same object was obtained in 1995 by M. C. Pedicchio in the case of exact Mal'tsev categories with coequalizers [8] and was further generalized to finitely cocomplete regular Mal'tsev categories in 2004 by D. Bourn [2].

In this work, we introduce a categorical-algebraic version of the ternary Bulatov commutator in the context of exact Mal'tsev categories with finite coproducts. We study its universal properties and some of its stability properties, extending the work of M. C. Pedicchio [8] and results of F. Borceux, D. Bourn and M. Gran [2, 3] in the binary case.

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