Finite difference methods for the reaction-wave-diffusion equation with distributed order in time

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In this talk we consider the reaction-wave-diffusion equation with distributed order in time:

$$\int_a^b c(\alpha)\frac{\partial^\alpha u(x,t)}{\partial t^\alpha}\, d\alpha = \frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t,u(x,t)), \quad t>0,\ 0\le x\le L, \tag{1}$$

with $0 \le a < b \le 2$ and the fractional derivative is given in the Caputo sense:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{-\alpha}\frac{\partial^n u(x,s)}{\partial s^n}\, ds, \tag{2}$$

where n is the smallest integer greater than or equal to α.
The function $c(\alpha)$ acting as weight for the order of differentiation and is such that $c(\alpha) \ge 0$ and $\displaystyle\int_a^b c(\alpha)\, d\alpha = C > 0$.
Will be presented a finite difference scheme to approximate the solution of (1), with a nonlinear source term, initial conditions given by

$$\begin{aligned} u(x,0) = g(x), \quad \text{if } \ a=0,\ b=1, \\ u(x,0) = g_0(x),\ \frac{\partial}{\partial t}u(x,0) = g_1(x) \quad \text{if } \ a=1,\ b=2, \end{aligned} \tag{3}$$

and the boundary conditions:

$$u(0,t) = \Phi_0(t), \quad u(L,t) = \Phi_L(t),\ t>0. \tag{4}$$

Results concerning the stability and convergence of the schemes are provided. Some examples with known analytical solutions are considered in order to show the efficiency of the numerical methods and illustrate the theoretical results.