Pretorsion theories and internal preorders

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The notion of *pretorsion theory* is a wide extension of the classical notion of torsion theory in an abelian category. In this talk we shall recall this notion and its basic properties, present a couple of new examples of pretorsion theories, and describe some monotone-light factorization systems naturally arising in these contexts.

The category $\mathsf{PreOrd}(\mathbb{C})$ of internal preorders in an exact category $\mathbb{C}$ contains the pretorsion theory $(\mathsf{Eq}(\mathbb{C}), \mathsf{ParOrd}(\mathbb{C}))$, where $\mathsf{Eq}(\mathbb{C})$ and $\mathsf{ParOrd}(\mathbb{C})$ are the full subcategories of internal equivalence relations and of internal partial orders in $\mathbb{C}$, respectively. We then observe that $\mathsf{ParOrd}(\mathbb{C})$ is a reflective subcategory of $\mathsf{PreOrd}(\mathbb{C})$ such that each component of the unit of the adjunction is a pullback-stable regular epimorphism. The reflector $\mathsf{PreOrd}(\mathbb{C}) \to \mathsf{ParOrd}(\mathbb{C})$ induces an admissible categorical Galois structure. When $\mathbb{C}$ is the category Set of sets, we show that this reflection induces a monotone-light factorization system in $\mathsf{PreOrd}(\mathsf{Set})$. A topological interpretation of our results in the category of Alexandroff-discrete spaces is also given, via the well-known isomorphism between this latter category and $\mathsf{PreOrd}(\mathsf{Set})$. Finally, some new observations about similar results in the category of preordered groups will be made.

This work is mainly based on a collaboration with Alberto Facchini and Carmelo Finocchiaro [1, 2]. The results concerning the category of preordered groups come from a recent work in collaboration with Aline Michel [3].

References

[1] A. Facchini, C. Finocchiaro and M. Gran, *A new Galois structure in the category of internal preorders*, Theory Appl. Categories, Vol. 35 (2020) 326-349.

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[3] M. Gran and A. Michel, *Torsion theories and coverings of preordered groups*, https://arxiv.org/abs/2005.07937, preprint (2020).