DOUBLE NEGATION AND OTHER SITES ON MONOIDES OF SEQUENCES RELATED TO THE CONVERGENCE

L. ESPAÑOL

In this talk, sites of the form $(M, \mathbf{I})$, where M is a monoid and $\mathbf{I}$ a Grothendieck topology on M, are considered. Working with spaces and convergent sequences, sites of this form were used in [3, 1, 2]. All of them are related with the site $(E, \mathbf{I}_{\neg\neg})$ formed by the monoid E of all monotone and injective maps $\mathbb{N} \to \mathbb{N}$, and its double negation topology. I will describe this particular site.

Given an extension of monoids $E \subseteq T$ and a Grothendieck topology $\mathbf{I}$ on E, we have a geometric morphism $sh(E, \mathbf{I}) \to \mathcal{T}$, where $\mathcal{T}$ is the topos of T-sets. The surjection-embedding factorization of this geometric morphism determines the factor topology on T [4].

Let T be the monoid of all continuous endomaps of the usual space $\mathbb{N}^+$, we have an extension $E \subseteq T$ by considering each $u \in E$ as a continuous map $\mathbb{N}^+ \to \mathbb{N}^+$ with $u(\infty) = \infty$. I will prove that the factor topology on T corresponding to the double negation topology on $\mathbb{E}$ is the canonical topology that defines Johnstone's topological topos.

References

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