UC|UP MATH PhD Program

Research Seminar Program

UC|UP Joint PhD Program in Mathematics

Abstracts

Pier Giorgio Basile: *A lax version of the Eilenberg-Moore adjunction* [1]
In Category Theory there is a well developed theory of monads, proved to be very useful for 1-dimensional universal algebra and beyond. The relation between adjunctions and monads was first noticed by Huber (*Homotopy Theory in General Categories*): every adjunction gives rise to a monad. Then, Eilenberg, Moore and Kleisli realized that every monad comes from an adjunction. In particular, Eilenberg and Moore (*Adjoint Functors and Triples*) realized that, for every monad $\mathbb{T}$, there is a terminal adjunction (called Eilenberg-Moore adjunction) which gives rise to $\mathbb{T}$. Category Theory can be also developed in a 2-dimensional case, that is, considering not only morphisms between objects but also morphisms (usually called 2-cells) between morphisms themselves. Thereby, one can study lax versions of the theory of monads. In the *pseudo* version, that is when we replace commutative diagrams by coherent invertible 2-cells, the relation between biadjunctions and pseudomonads has been investigated by F. Lucatelli Nunes in the paper *On Biadjoint Triangles* as a consequence of the coherent approach to pseudomonads of S. Lack. The next step consists of studying the *lax* notion of monads, in which the associativity and identity works only up to coherent (not necessarily invertible) 2-cells. In this talk we present a work in progress where we try to generalize to the lax-context the classical result of Eilenberg-Moore. For this purpose, having in mind the notion of lax extension of monads introduced and studied in the context of *Monoidal Topology (Metric, topology and multicategory: a common approach - M.M. Clementino and W. Tholen)*, we use a generalization of Gray's lax-adjunction (see the monograph *Formal Category Theory*). Then, we show some steps of the construction leading to the positive answer.

Alberto José Hernández Alvarado: *The Quotient Module, Coring Depth and Factorisation Algebras*
In this conference I will be reviewing the main aspects of my thesis dissertation. I will introduce the notion of depth of a ring extension $B \subseteq A$ and give several examples as well as important results of recent years. I will then consider a finite dimensional Hopf algebra extension $R \subseteq H$ and its quotient module $Q := H/R^+H$ and show that the depth of such an extension is intrinsically connected to the representation ring of H, $A(H)$. In particular, we will see that finite depth of the extension is equivalent to the quotient module Q being algebraic in $A(H)$. Next I will introduce entwining structures and use them to show that a certain extension of crossed product algebras is a Galois coring and use that to give a theoretical explanation for a result of S. Danz (2011). Finally, I will discuss factorisation algebras and their roll in depth, in particular a result on the depth of a Hopf algebra H in its generalised factorised smash product with Q^{*op}.

Fernando Lucatelli Nunes: *Kan construction of adjunctions*
I will talk about a basic procedure of constructing adjunctions, sometimes called Kan construction/adjunction. In the first part of the talk, I will construct abstractly such adjunctions via colimits. In the second part, we give some elementary examples: fundamental groupoid, sheaves, etc. We assume elementary knowledge of basic category theory (definition of categories, colimits and Yoneda embedding).

Antonio Macchia: *Proper divisibility as a partially ordered set* [2]
We define the order relation given by the proper divisibility of monomials, inspired by the definition of the Buchberger graph of a monomial ideal. From this order relation we obtain a new class of posets. Surprisingly, the order complexes of these posets are homologically non-trivial. We prove that these posets are dual CL-shellable, we completely describe their homology (with integer coefficients) and we compute their Euler characteristic. Moreover this order relation gives the first example of a dual CL-shellable poset that is not CL-shellable.

[1] joint work with Fernando Lucatelli Nunes
[2] joint work with Davide Bolognini, Emanuele Ventura and Volkmar Welker